

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 3987

Roll No.

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B.Tech.**(SEMESTER-IV) THEORY EXAMINATION, 2011-12****MATHEMATICS – III****Time : 3 Hours]****[Total Marks : 100**

Note : Attempt questions from each Section as indicated. The symbols have their usual meaning. Provide statistical tables which are required by students.

Section – AAttempt **all** parts of this question. Each part carries **2** marks :**10 × 2 = 20**

- 1, (a) If $f(z) = u + iv$ is analytic, then show that the family of curves $u(x, y) = c_1$ and $v(x, y) = c_2$ are mutually orthogonal.
- (b) Define removable and essential singular points with example.
- (c) Define the coefficients of Skewness and Kurtosis.
- (d) What is the total probability theorem ?
- (e) Explain in brief Null and Alternative hypotheses.
- (f) Define coefficient of contingency.
- (g) Isolate the roots of the equation $x^3 - 4x + 1 = 0$.
- (h) Verify that $\nabla E \equiv \Delta$.
- (i) What do you mean by numerical differentiation ? Explain in brief.
- (j) Let $I = \int_{x_0}^{x_3} f(x)dx$, where $f(x)$ is a third degree polynomial. Write the formula you

will like to use to find the approximate value of I . It is given that the data are equispaced.

Section – B

2. Attempt any **three** parts of this question.

3 × 10 = 30

- (a) Verify Cauchy's theorem by integrating $\exp(iz)$ along the boundary of the triangle with vertices at the points $1 + i$, $-1 + i$ and $-1 - i$.
- (b) Find all four central moments and discuss skewness and kurtosis and also Karl Pearson skewness for the frequency distribution given in the following table :

Range of expenditure (in ₹ 100 per month)	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12
No. of families	38	292	389	212	69

- (c) A manufacturer claimed that at least 95% of the equipments which he supplied to a factory conformed to the specifications. An examination of a sample of 200 pieces of equipments revealed that 18 were faulty. Test this claim at a significant level of (i) 0.05 and (ii) 0.01.
- (d) Show that the Newton-Raphson Method has second order convergence.
- (e) Solve the following system using Crout's decomposition method :

$$3x - y + 2z = 12$$

$$x + 2y + 3z = 11$$

$$2x - 2y - z = 2$$

Section – C

All questions of this section are compulsory. Attempt any **two** parts from each question.

5 × 10 = 50

3. (a) Determine p such that the function

$$f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1}\left(\frac{px}{y}\right) \text{ is an analytic function. Also find } f'(z).$$

- (b) Find Laurent series expansion of

$$f(z) = \frac{4z-1}{z^4-1}$$

about the point $z = 0$.

- (c) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4 \cos \theta} d\theta$.

4. (a) Fit a parabola of the form $y = a + bx + cx^2$ to the data

x	1	2	3	4
y	1.7	1.8	2.3	3.2

by the method of least squares.

- (b) If θ is the acute angle between the two regression lines in case of two variables x and y , show that

$$\tan \theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2},$$

where r , σ_x , σ_y have their usual meanings. Explain the significance of the formula when $r = 0$ and $r = \pm 1$.

- (c) Out of 800 families with 5 children each, how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys? Assume equal probabilities for boys and girls.

5. (a) Fit a binomial distribution to the data given in the following table :

x	0	1	2	3	4
f	24	41	28	5	2

- (b) The number of scooter accidents per month during a year in a certain town were as follows :

12, 8, 20, 2, 14, 10, 15, 6, 9, 4, 7, 13

Are these frequencies in agreement with the belief that the accident conditions were the same during the whole year?

- (c) \bar{X} and R values for 10 sub-groups of 5 readings are given in the following table.

Determine the control limits for \bar{X} and R charts for future use, eliminating all the out of control points :

Sub-group numbers	1	2	3	4	5	6	7
\bar{X}	34.0	31.6	30.8	33.0	35.0	32.2	33.0
R	4	4	2	3	5	2	5

Sub-group numbers	8	9	10
\bar{X}	32.6	33.8	37.8
R	13	19	6

6. (a) Find the root of $2 \sin x - 2x + 1 = 0$ correct to five significant digits with initial approximation $x_0 = 1.0$.

- (b) Estimate the values of a and b in the following table :

X	10	15	20	25	30	35
f(x)	43	a	29	32	b	77

- (c) The population of a town in decennial census is as under. Estimate the population for the year 1955 :

Year	1921	1931	1941	1951	1961
Population (in lac)	46	66	81	93	101

7. (a) Find $\frac{d}{dx}(J_0)$ at $x = 0.1$ from the following table :

x	0.0	0.1	0.2	0.3	0.4
$J_0(x)$	1.0000	0.9975	0.9900	0.9776	0.9604

Also find $\frac{d^2}{dx^2}(J_0)$ at $x = 0.1$.

- (b) Evaluate the integral

$$\int_0^{2\pi} e^{-t} \sin(10t) dt$$

using Simpson's 3/8 rule.

- (c) Apply Picard's method to find the solution of the initial value problem

$$\frac{dy}{dx} = y - x, y(0) = 2$$

Show that the iterative solution approaches the exact solution.